

# TLM Synthesis of Microwave Structures Using Time Reversal

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## Abstract

This paper presents a novel numerical synthesis technique based on the reversal of the TLM process in time. It allows the designer to generate the geometry of a passive circuit from its desired frequency response using alternate forward and backward time domain simulation. The essential steps of the procedure are explained and validated using, as an example, the synthesis of an inductive obstacle in a waveguide.

## 1 Introduction

Traditional methods for synthesis of microwave structures consist of repeated analyses combined with an optimization strategy. Depending on the nature of the problem, a large number of analysis cycles are necessary to achieve satisfactory convergence of the design process. Although they demonstrate very good performance in the frequency domain, those successive analyses would be very wasteful in the time domain since each analysis requires the transient build-up of the field from an impulsive excitation. Thus, when using a time domain method, different approaches should be explored that take advantage of the time dimension. The principles of a new technique based on time reversal have recently been introduced by Sorrentino *et al.* [1]. This new procedure performs a time domain synthesis of microwave structures with the Transmission-Line Matrix method (TLM).

The main advantage of time domain methods resides in the ability to work with a large bandwidth signal and the possibility to include transient phenomena, which are both important in the time domain synthesis of a microwave structure. In this paper, the new synthesis procedure will be described in detail and applied to a simple example so as to not obscure the essence of the method by the complexity of the structure. Finally, results will be discussed and compared with existing design methods.

## 2 Reversibility of Time in the TLM Method

The Transmission-Line Matrix (TLM) method was first introduced by Johns and Beurler [2]. It is a discrete time and space sampling method for modeling the propagation of

electromagnetic waves. The propagation space is represented by a mesh of interconnected transmission-lines. The well-known two-dimensional TLM shunt node is represented schematically in Fig. 1.

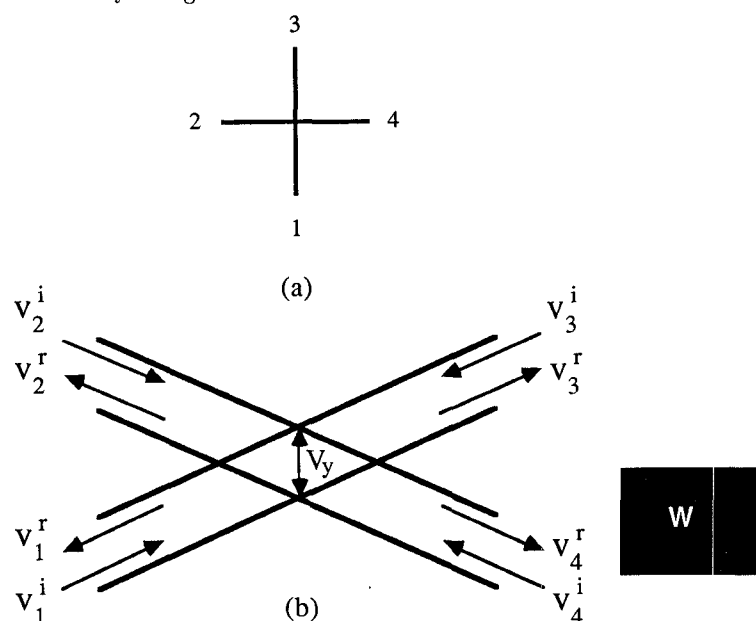


Fig. 1 2D-TLM shunt node  
(a) schematic (b) three-dimensional view

For two-dimensional homogeneous regions, the voltage impulses  $V_n^i$  incident on each line are scattered according to the node scattering matrix

$$\begin{bmatrix} V_1^r \\ V_2^r \\ V_3^r \\ V_4^r \end{bmatrix}_{k+1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} V_1^i \\ V_2^i \\ V_3^i \\ V_4^i \end{bmatrix}_k \quad (1)$$

Furthermore, each scattered impulse  $V_n^r$  at a given node becomes an incident impulse  $V_n^i$  at the adjacent node. The total node voltage  $V_y$  at the instant of scattering is

$$V_y = \frac{1}{2} \sum_{n=1}^4 V_n^i \quad (2)$$

It has been shown by Sorrentino *et al.* [1] that the TLM process is reversible due to the fact that the scattering matrix is equal to its inverse,

$$S^{-1}S = I \quad (3)$$

This property of the TLM scattering matrix implies that by reversing the process without changing the algorithm, one can reconstruct a source distribution from a known field solution by reversing the TLM process in time. This is also true for the three-dimensional distributed and condensed node algorithms. An extensive list of references on these algorithms is given in [3].

## 2 Reconstruction Technique Using Time Reversal

The essence of the time reversal synthesis will be explained and demonstrated for the reconstruction of a simple discontinuity in a parallel plate waveguide. This configuration can propagate a TEM wave, and possesses a complete spectrum of well-defined high order modes.

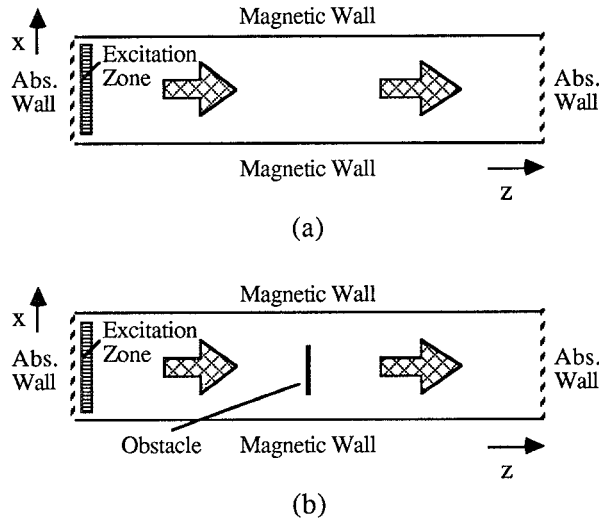


Fig. 2 Parallel plate waveguide arrangement .  
(a) Empty waveguide yielding the homogeneous solution,  
(b) Waveguide containing the obstacle and yielding the total solution.

Fig. 2 shows the parallel plate waveguide without (a) and with (b) the obstacle. Both structures are terminated at both ends by an absorbing boundary and excited in the linear excitation zone by a Gaussian impulse, a signal with a limited bandwidth such that the travelling waveform is not noticeably distorted due to velocity dispersion. The obstacle is a septum consisting of a magnetic wall inserted into the center of the parallel plate waveguide. The following procedure consists of three stages :

1. Forward analysis of the empty waveguide to obtain the homogeneous field solution in the empty waveguide.
2. Forward analysis of the loaded waveguide to obtain the total field solution which is the sum of the homogeneous solution and the scattered field (particular solution).
3. Backward synthesis to reconstruct the topology of the scattering obstacle.

In the first stage, a time-sampled Gaussian impulse is injected into the empty waveguide section, and the resultant impulses incident upon both absorbing boundaries are picked up and stored. We refer to these homogeneous impulses at each boundary node as  $\Phi_{left}^h(i, k)$  and  $\Phi_{right}^h(i, k)$ .

The index “ $i$ ” is related to the position in the transversal direction, while the index “ $k$ ” represents the discrete time step.

In the second stage, the same function is stored with the discontinuity in place, yielding the impulse responses  $\Phi_{left}^t(i, k)$  and  $\Phi_{right}^t(i, k)$ .

Note that the number  $k$  of time steps must be the same in both cases. Finally, in the third step, both impulse responses are subtracted from each other as follows, yielding the particular solutions

$$\Phi_{left}^p(i, k) = \Phi_{left}^t(i, k) - \Phi_{left}^h(i, k), \quad (4)$$

and

$$\Phi_{right}^p(i, k) = \Phi_{right}^t(i, k) - \Phi_{right}^h(i, k). \quad (5)$$

These functions are then injected in the reverse time sequence at both sides of the empty waveguide. The same number of iterations is performed as in the forward simulations. Note that for the analysis, the number of iterations must be long enough to let the field vanish completely in the waveguide because it is assumed that there is no field in it when the inverse simulation is started.

At every computational step of the synthesis, the absolute value of the Poynting vector is displayed at each node inside the structure according to a color map and updated if the current value is larger than the maximum previous value. The Poynting vector value is computed using the following expression :

$$|P(i, j)| = |E_y(i, j)| \sqrt{|H_x(i, j)|^2 + |H_z(i, j)|^2}, \quad (6)$$

where the indices “ $i$ ” and “ $j$ ” indicate the position in the waveguide. Therefore, the final picture of the distribution represents the maximum value that occurred at each node during the entire process. In this manner, the shape of the obstacle can be reconstructed. As an example, Fig. 3(a) shows the final result from which the septum can be reconstructed. The valley in the distribution of the magnitude of the Poynting vector describes the location and

the shape of the septum. An algorithm which extracts the position of this valley is used to determine the shape of the obstacle. Fig. 3(b) shows the image of the reconstructed obstacle.

Obviously, the resolution depends on the relative density of the mesh and the resulting resolution of the Gaussian excitation waveform.

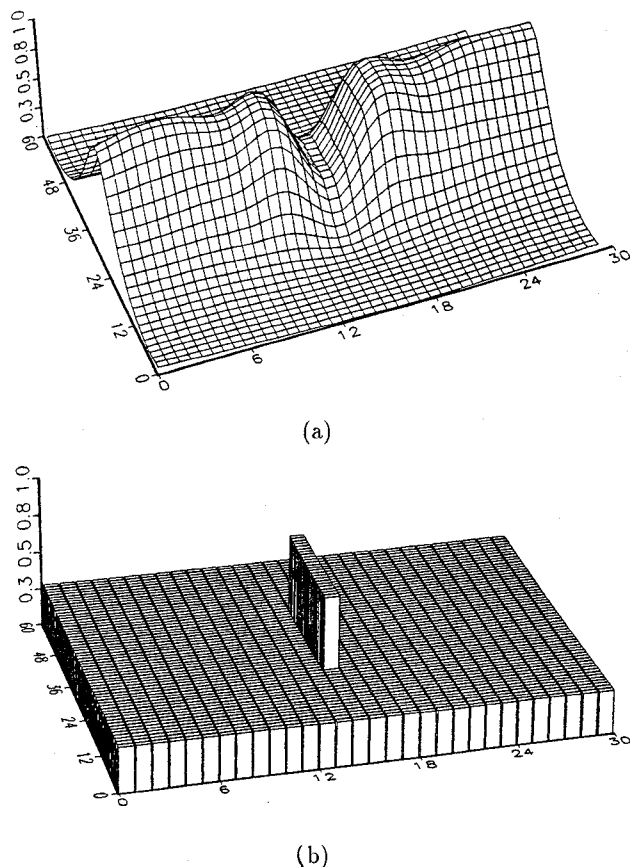


Fig. 3 Reconstruction of a metallic septum in a parallel plate waveguide. (a) Distribution of the maximum magnitude of the Poynting vector after the injection of the reversed impulse response. (b) Geometry and position of the septum extracted from (a).

#### 4 Complete Synthesis Procedure

A realistic synthesis does not start with an analysis of the desired structure as in the previous demonstration, but rather from a desired time or frequency response, the latter being the more frequently encountered case. However, such a frequency response usually does not contain information for frequencies considerably higher than the upper band limit of interest. For example, the specifications for an inductive iris are given only for the dominant mode, and there is

no information about the distribution of energy in higher order modes. In other words, there is no information about the transversal distribution of the fields in the reference planes. Nevertheless, this information is essential for the reconstruction of the obstacle.

Hence, this information must be obtained somehow. To this end, a first forward TLM analysis is performed of an obstacle having approximately the desired dimensions. These can be found in many cases from closed-form expressions, given in the literature, which link the dimensions of discontinuities to their equivalent lumped element circuits. The dominant mode content of this first response  $\Phi_{left}^t(i, k)$  and  $\Phi_{right}^t(i, k)$  is then extracted and replaced by the desired dominant mode content in the frequency domain. The modified total response is then converted back into the time domain and, reduced by the homogeneous response of the empty waveguide, reinjected into the computational domain in the inverse time sequence. This procedure will now be described in more detail.

Let the impulse response of the approximated obstacle be  $f_1(t)$  with its Fourier transform  $F_1(\omega)$ . The latter is most likely different from the desired frequency response  $F_2(\omega)$  which is usually defined over a limited lower frequency range and in the dominant mode only. We can thus modify  $F_1(\omega)$  so that it is identical to  $F_2(\omega)$  in the bandwidth of interest, and for the dominant mode of propagation. Now the modified response  $F_1'(\omega)$  must be converted to a time domain signal for reinjection into the empty waveguide. In this process, the linear property of the Fourier transform is used.

Since  $F_1(\omega) - F_1'(\omega)$  is limited to the frequency band in which only the dominant mode can propagate, the transverse field distribution of the corresponding time function  $f_1(t) - f_1'(t)$  is known. The time domain signal of the modified frequency response is thus

$$f_1'(t) = f_1(t) - \underbrace{\mathcal{F}^{-1}[F_1(\omega) - F_2(\omega)]}_{\text{known transversal distribution}} \quad (7)$$

This new time domain impulse response  $f_1'(t)$  approximates the wanted impulse response since it contains, for the bandwidth of interest, the dominant mode response of the obstacle to be synthesized.

Finally, the difference between  $f_1'(t)$  and the homogeneous response is reinjected into the empty structure, yielding an image of the synthesized obstacle as described in the previous section. This result represents an improvement over the initial guess and in many cases, is already acceptable as a final result. However, if a new analysis still yields an unsatisfactory answer, the same sequence is repeated until it converges. Fig. 4 shows a flow chart for this synthesis technique.

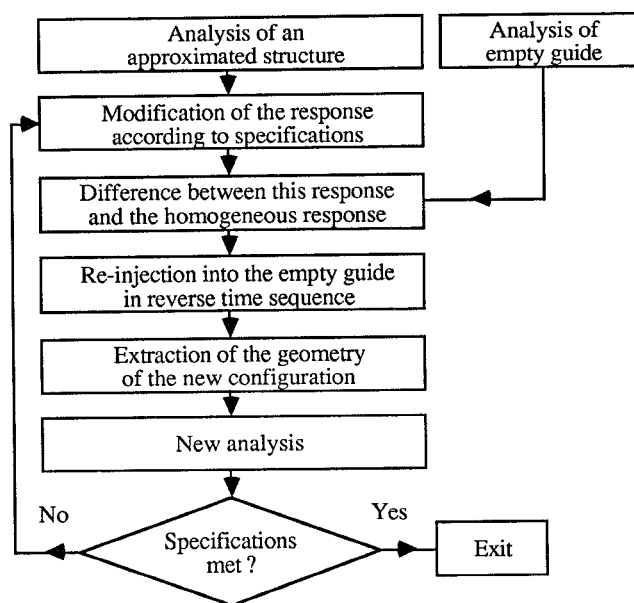


Fig. 4 Flow chart of the proposed synthesis procedure

## 5 Discussion

The proposed new technique has shown promising results. But some additional details must be explained. First, if the process is followed as described above, only the left part of the obstacle will appear. This is due to the fact that induced sources appear mainly on the left side where the incident wave front hits. However, a simulation with illumination from the right side can also be performed to get the image of the other part of the obstacle, or both can be combined in a single process involving even excitation from both sides.

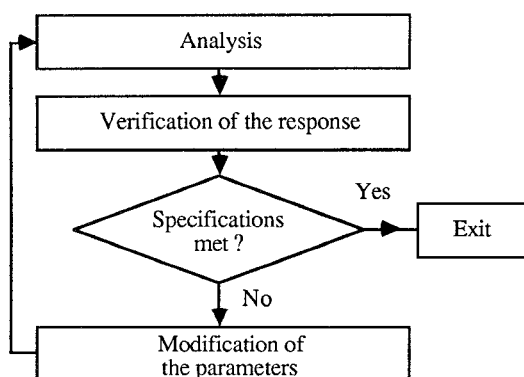


Fig. 5 Flow chart of traditional CAD procedure

Also, current design techniques are based on optimization procedures [4]. Fig. 5 shows a simplified algorithm for such

techniques. Usually, the number of variables in optimization procedures must be small to ensure fast convergence of the process. However, the proposed technique is independent of the shape of the obstacle. Thus, for a more complex structure which would require an important number of variables, the synthesis procedure using time reversal would be more efficient than an optimization procedure. The performances of the new method depends mainly on the mesh density and the quality of the initial guess for the dimensions of the obstacle.

## 6 Conclusion

A new technique for the time domain synthesis of microwave structures has been presented. Based on the principles introduced by Sorrentino, So and Hoefer [1], this new efficient algorithm based on the Transmission-Line Matrix method has yielded good results for simple scatterers inside a waveguide. It has been shown that the required information on higher order modes (which is not provided by the design specifications) can be generated by a forward analysis of an approximated structure. It is at this point that the designer can make a choice among several acceptable solutions. For example, an inductive obstacle could be realised in many possible forms, such as rectangular or circular posts, longitudinal or transverse septa, irises, etc. This distinguishes design from the inverse scattering where a specific unknown scatterer must be reconstructed. Future research will be aimed at the development of more general procedures for the synthesis of more complex geometries such as filters, couplers and hybrid junctions.

## References

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